Experimental observation of perturbation-induced intermittency in the dynamics of a loss-modulated CO_2 Laser

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We show experimentally in the dynamics of a periodically perturbed CO_2 laser with modulated losses that a different type of intermittency (the ''breathing effect'') [Z. Qu, G. Hu, G. Yang, and G. Qin, Phys. Rev. Lett. **74**, 1736 (1995)], which is characterized by a regular alternation between periodic and chaotic behaviors, can be created by different methods. In addition to the usual way, (i) by suppressing chaos by weak perturbations, we have produced the breathing effect (ii) by destabilizing periodic stable orbits created at a period-doubling bifurcation and (iii) by means of large amplitude perturbations in the bistability domain between coexisting attractors. In addition, we also observed the crisis of a strange attractor induced by periodic perturbations and phase effects in the nonfeedback control of laser dynamics. [S1063-651X(96)07409-0]

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Recently [1–3], in the context of the nonfeedback control of chaos in nonautonomous systems by periodical perturbations, it has been shown that small deviations of the perturbation frequency from a parametric resonance result in a regular alternation between periodic and chaotic behaviors. In Ref. [3] such a type of temporal behavior has been analyzed in detail for the Duffing equation and identified as a different type of intermittency, called the ''breathing effect'' in dynamical systems. This effect is caused by the quasistatic periodic drift in the phase coming from the small detuning of the perturbation frequency with respect to parametric resonance in the system, as already pointed out in Ref. [2].

In this paper we show experimentally that this different type of intermittency, due to near-resonant (i.e., close to parametric resonance) periodic perturbations, is a more general phenomenon than it was experimentally observed in Refs. [1,2]. We demonstrate that such a kind of temporal behavior can occur in different experimental conditions: not only (i) induced by weak perturbations close to subharmonic resonance when the unperturbed system is initially in a chaotic state, as already demonstrated [1,2], but also (ii) induced by perturbations when the system is initially in one of the periodic stable states created at a period-doubling bifurcation and (iii) induced by relatively large perturbations close to the frequency f/n (where f is the driving frequency and $n=2,3,\ldots$) in the bistability domain between period-1 and period-n branches.

We also found that in some cases, after symmetry-breaking bifurcation, this intermittency is characterized by a disappearance in the spectra of the spectral components corresponding to the primary period-doubling bifurcation. We also demonstrate that large-amplitude resonant perturbations (in contrast to the weak ones normally used in suppressing chaos) can induce crisis of strange attractors. In addition, we show for different initial dynamical states that the phase of a resonant perturbation of a given amplitude plays a key role in the stabilization or destabilization of the system.

Experiments have been carried out on a single-mode frequency-stabilized CO_2 laser with two acousto-optic modulators placed inside the cavity, as it has been described earlier [4]. The sinusoidally varying driving voltage $V_d(t)=V_d\cos(2\pi f_d t)$ was applied to one modulator and the perturbing signal $V_s=V_s\cos(2\pi f_s t)$ was applied to the other modulator. Here f_d and f_s are the driving and perturbing frequencies, respectively, and V_d and V_s are their amplitudes. These modulators have different efficiency with respect to the applied voltage so that the driving modulator was able to produce the loss modulation several times more efficiently than the perturbation modulator. In our experiments $f_s=f_d/n+\delta$, where $n=2,3$ and δ is a small detuning. The maximal deviations of the detuning δ from its mean value did not exceed $\pm 2\%$. The laser output intensity was monitored by a Hg-Cd-Te detector with a time resolution of 50 ns and a digital oscilloscope with memory capacity of 2000 points coupled to a PC/AT486 computer. In all our experiments we used the technique of periodic sampling the laser intensity with the frequency f_d of the modulation and a constant phase with respect to this harmonic modulation. While in Ref. [4] we focused on parametric effects near period doubling of perturbing signals with large detunings $(0.01f_d/2\leqslant\delta\leqslant 0.1f_d/2)$, we concentrate here on the case of small detunings $(\delta<10^{-3}f_d/2)$.

First let us consider the effects of periodic perturbations at a frequency close to $f_d/2$ on the dynamics of the system in the chaotic state. For this case the modulation frequency (f_d = 143.5 kHz) and the bifurcation parameter (V_d = 10 V) were chosen so that two attractors, associated with period-1

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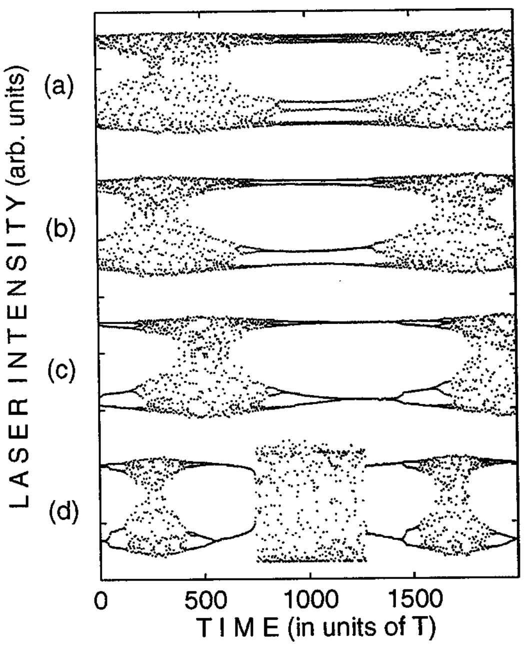

FIG. 1. Experimental CO_2 laser responses (in arbitrary units) versus time (in units of the modulation period T) for different values of the perturbation amplitude V_s. The unperturbed system (with $V_s = 0$) is in a chaotic state. The laser intensity is sampled with the modulation period T. (a) $V_s = 0.62$ V, (b) $V_s = 0.8$ V, (c) $V_s = 1.08$ V, and (d) $V_s = 4.7$ V. Here the modulation frequency is $f_d = 143.5$ kHz and the detuning of f_s from $f_d/2$ is $\delta = 50$ Hz.

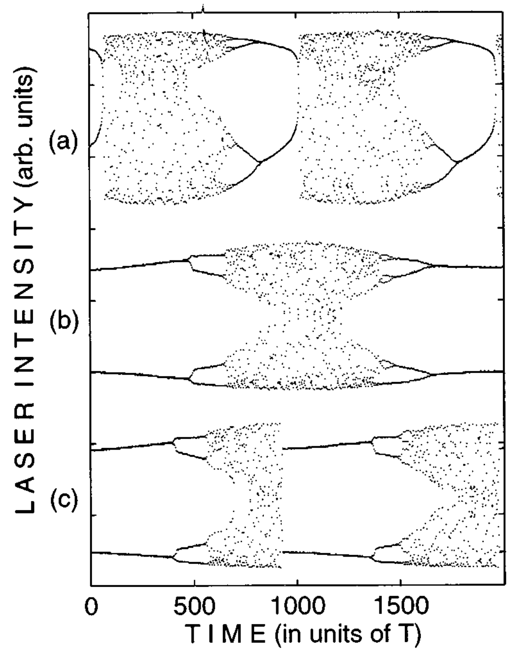

FIG. 2. Experimental CO_2 laser intensity (in arbitrary units) showing the destabilizing effect of near-resonant perturbations with different amplitude V_s when the unperturbed system (with $V_s = 0$) is in a $2T$ stable orbit. The laser intensity is sampled with the modulation period T. (a) $V_d = 6.7$ V, $V_s = 10.2$ V; (b) $V_d = 8$ V, $V_s = 4.5$ V; and (c) $V_d = 8.25$ V, $V_s = 4.5$ V. Here the modulation frequency is $f_d = 148.5$ kHz and $f_s = 74.17$ kHz.

chaos and period-3 regimes, can coexist. The experimental results are represented in Fig. 1, where we show stroboscopic data recording of the CO_2 laser intensity as a function of time in units of the modulation period T ($T = 1/f_d$) for different values of the perturbation amplitude V_s. It is clearly seen that the effect of near-resonant perturbations on the stability of the system is not static but changes periodically in time with a long period that is determined by δ. Increasing the perturbation amplitude V_s, the system regularly drifts from chaos to stable orbits of $8T$ [Fig. 1(a)], $4T$ [Fig. 1(b)], and $2T$ [Fig. 1(c)] through inverse cascades of period doubling. Here $1\frac{1}{2}$ periods of the alternation $T_a = 1/2\delta$ are shown. This period T_a corresponds approximately to 1400 points in Fig. 1. For some critical value of the perturbation amplitude ($V_c = 3.7$ V) the temporal behavior changes abruptly. This critical value V_c is larger than the one corresponding to taming chaos in Fig. 1(c) approximately by only a factor 3.5. In Fig. 1(d) we show the case when the perturbation amplitude is larger than the critical value V_c by approximately 1.3 times. In this case there is a regular appearance of period-1 branch chaos, stable periodic orbits, and chaotic behavior connected with the interaction of a period-3 branch. As the perturbation amplitude value is further increased above the critical value V_c, the time spent being chaotic in this state also increases. We found in the range of perturbation amplitude used in our experiments that this time (denoted here as τ) obeys a simple scaling law $\tau \propto (V_s - V_c)^\alpha$, where $\alpha = 0.34 \pm 0.03$.

We identified the temporal behaviors represented in Fig. 1 as a different type of intermittency induced by periodic perturbations in dynamical systems. Results similar to those in Fig. 1(b) were reported earlier by Meucci *et al.* [2], who used linear sweeping of the phase of the resonant perturbations. The mechanism of this effect has been clearly described theoretically in Refs. [2,3] and is called the breathing effect [3].

The second way to obtain perturbation-induced intermittency is by destabilization of stable periodic orbits that appear at period-doubling bifurcations. The destabilizing effect of near-resonant perturbations has been observed experimentally by Vohra *et al.* [5] but in their work they did not report about the temporal behavior of the system. Some experimental results illustrating intermittency resulting from such a destabilization are shown in Fig. 2. This figure corresponds to the case when the unperturbed system was initially in a $2T$ periodic regime ($V_d \geqslant 6.7$ V) far from the first threshold of period doubling (the threshold value of the first period doubling $V_{d1} = 3.3$ V). The perturbation amplitudes are practically the same as the ones for which suppression of chaos can be obtained for higher values of the driving amplitudes at the frequency $f_d = 148.5$ kHz. It is seen that the near-resonant perturbation destabilizes the system so that it periodically drifts from the $2T$ regime to chaos by an inverse period-doubling cascade [Fig. 2(a)]. Figures 2(b) and 2(c) show the cases for some other driving and perturbing amplitudes. The bifurcation parameters in these two figures differ by a few percent. It is seen that when the control parameter slightly increases, the temporal behavior changes suddenly. While in Fig. 2(b) there are direct and inverse period-doubling cascades to chaos, in Fig. 2(c) there are only direct cascades from $2T$ to chaos. This change of temporal behavior corresponds to a symmetry-breaking bifurcation that oc-

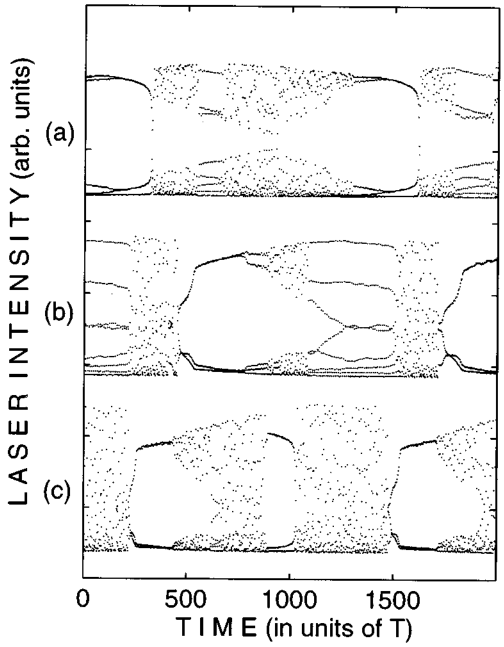

FIG. 3. Experimental stroboscopic data of CO_2 laser intensity (in arbitrary units) showing the effect of the perturbation at $f_d/3+\delta$ for different values of the bifurcation parameter V_d in the bistability domain between period-1 and period-3 branches. The laser intensity is sampled with the modulation period T. (a) $V_d = 8.5$ V, (b) $V_d = 9.25$ V, and (c) $V_d = 10.5$ V. Here the modulation frequency is $f_d = 184.67$ kHz and $f_s = 61.58$ kHz. The perturbation amplitude is $V_s = 11$ V.

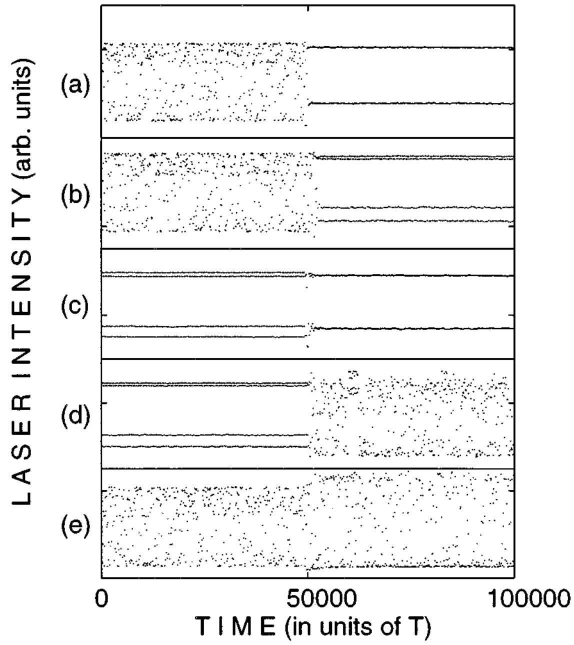

FIG. 4. Experimental stroboscopic data of CO_2 laser intensity (in arbitrary units) showing the effect of resonant perturbations at $f_d/2$ with different amplitude V_s and the same phase with respect to the driving frequency f_d [(a), (b), and (e) $V_d = 10$ V and the unperturbed initial state is chaotic] and with the same amplitude V_s and two values of the phase differing approximately by $\pi/2$ [(c) and (d) V_d=8.5 and the unperturbed initial state is a $4T$ orbit]. The laser intensity is sampled with the time period $101T$. (a) $V_s = 0.62$ V; (b) $V_s = 0.8$ V; (c) $V_s = 0.8$ V; (d) V_d=8.5, $V_s = 0.8$ V; and (e) $V_s = 4.7$ V. Here the modulation frequency is $f_d = 143.5$ kHz.

"curs in the system when the amplitude of periodic perturbations reaches some critical value. In this case the laser response is characterized by the disappearance of the primary period doubling (the response at $f_d/2$) and all even spectral components at $f=f_d/2\pm m\delta$, where $m=2,4,\ldots$. The same effect is found when increasing the perturbation amplitude in the conditions of Fig. 2(a). This effect is of a similar nature as the one theoretically considered in Ref. [6], where the suppression of period doubling and the shift of the bifurcation point due to near-resonant perturbations as well as a number of related nonlinear parametric effects were predicted from an analysis of a normal form equation. Thus we can state that near-resonant perturbations can simultaneously destabilize and stabilize the system in the sense that they induce chaotic behavior with suppressed primary period doubling [7].

One more way to obtain perturbation-induced intermittency is by involving isolated branches (which can exist in nonautonomous systems) in the dynamics. Figure 3 shows some examples with an isolated period-3 branch for different values of the bifurcation parameter and a constant perturbation amplitude. In this case the modulation frequency (f_d = 184.6 kHz) is slightly less than three times the relaxation-oscillation one ($f_r = 68$ kHz) and the perturbation frequency $f_s=f_d/3+\delta$ ($f_s = 61.58$ kHz). Using the technique of short-lived loss perturbations [8], we found that period-1 and period-3 branches coexist for this modulation frequency. It is clearly seen that the system drifts between the $3T$ periodic orbit and chaos showing inverse [Fig. 3(a)] and direct [Fig. 3(b)] period-doubling cascades of the $3T$ branch. Figure 3(c) demonstrates a more complex example of intermittent behavior. It should be noted that involving isolated branches by periodic perturbations occurs when the perturbation amplitude reaches some critical value [7]. The time length T_a after which the qualitative dynamic behavior repeats periodically is approximately $T_a=1/3\delta$ for the conditions of Fig. 3.

Owing to the small detuning used (in all our experiments $\delta<10^{-3}f_d/2$), which means quasistatic changes in the phase, the data represented in Figs. 1–3 can be considered also as phase bifurcation diagrams for given amplitudes of resonant perturbations where the phase acts as the control parameter. This allows, for example, one to find experimentally the needed conditions for robust taming of chaos by resonant perturbations in the same manner as it has been experimentally performed in Ref. [2] with linear sweeping of the phase of the resonant perturbations. In Figs. 4(a) and 4(b) we show a stabilization of $2T$ and $4T$ orbits by resonant perturbation at $f_d/2$. In our experimental conditions the robust stabilization was observed over many hours. Moreover, one more important result follows immediately from Figs. 2 and 3: that the phase of the resonant perturbation can have a determining effect on the dynamics of the system not only in chaos [2,3] but for any dynamical states. Changing the phase of the given resonant perturbations, we can easily shift period-doubling and saddle-node bifurcation points and obtain any desired dynamical state up to chaotic behavior. In Figs. 4(c) and 4(d) we show the suppression of a $4T$ orbit and chaos induced by resonant perturbation at $f_d/2$, respectively, when the system was initially in a $4T$ orbit. In addition to phase control of chaos [2,3], we can consider such a type of control as phase control of dynamical systems.

Finally, in Fig. 4(e) we show the crisis of a strange attractor induced by resonant perturbation of a relatively large amplitude in comparison with the ones used in suppressing chaos. By analogy with a noise-induced crisis we called it a perturbation-induced crisis. Our conclusion is based on the following. (i) Above some critical value of the perturbation amplitude V_c, the amplitudes of the laser response suddenly increase. In reality, these amplitudes are larger than shown in Fig. 4(e) because of the sampling made with the constant phase tuned to maximal peaks of period-1 with respect to the phase of the modulation frequency. (ii) By a comparison of the intensity return maps in Figs. 5(a) and 5(b), it is seen that many points of the laser response lie outside of the pattern of points corresponding to the unperturbed chaotic attractor. (iii) For given experimental conditions we found experimentally the coexistence of two attractors of period-1 and period-3 for the same set of the laser parameters using the technique of short-lived loss perturbations as described in Ref. [8].

In the context of the anticontrol or maintenance of chaos, which was experimentally implemented in a magneto-mechanical system [9], the effect of perturbation-induced crisis can be very useful because it allows one to obtain an increased complexity of the motion in the system by very simple methods, i.e., by applying it to the system resonant periodic perturbations with the needed amplitude and phase. Moreover, occasionally switching on and off the resonant perturbations, we can easily make intermittent this increasing of complexity.

In conclusion, we have experimentally shown that perturbation-induced intermittency can be obtained in different ways. In addition, we have also demonstrated that resonant perturbation can induce crisis of strange attractors in the bistability domain of coexisting period-1 and period-3 branches. The data presented here can be also considered as phase bifurcation diagrams, allowing one to easily choose the needed amplitude and phase of a resonant perturbation in order to obtain any desired periodic regime starting not only from chaos [2,3] but from any periodic orbit.

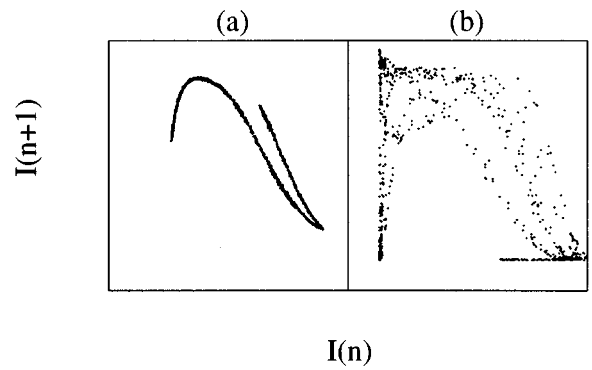

FIG. 5. Intensity return maps corresponding to Fig. 4(e): (a) the initial unperturbed chaotic state ($V_s = 0$) and (b) after switching on the resonant perturbation at $f_d/2$ ($V_s = 4.7$ V). Here $f_d = 143.5$ kHz.

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